SPECIFIC FEATURES OF THE ACOUSTIC-GRAVITY WAVE PROPAGATION IN THE SOLAR CHROMOSPHERE

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Abstract. Some new features of the acoustic-gravity wave propagation from the sources at the photospheric heights through a nonisothermic solar chromosphere are examined. Within the framework of the plane-layered model of the atmosphere some properties of the wave perturbations near the height at which the horizontal phase wave velocity coincides with the local sound velocity are studied. At this height, a resonance singularity in the pressure disturbance occurs and above the resonance level the wave field is absent. The results of numerical calculation of the wave field by means of the full-wave numerical model are given for the experimentally known altitude temperature profile. The conclusion is made that a relatively high-frequency acoustic branch of acoustic-gravity waves with periods less than two minutes does not contribute to the vertical energy flux through the chromosphere.

Key words: chromosphere - acoustic gravity waves - filtration - nonisothermal atmosphere

1. Introduction

The peculiarities of acoustic-gravity waves propagation in the solar atmosphere are of interest to researchers from the point of view of the possibility of the energy transfer to the upper solar atmosphere (see, e.g., Kaplan \textit{et al.}, 1977; Gibson, 1973; Aschwanden, 2004). This question remains open until now (Fossum and Carlsson, 2005). One of the main unanswered questions in solar physics is why the solar corona is hotter than the stellar surface. Eighty years ago Schwarzschild and Biermann independently proposed that comparatively high frequency acoustic waves play an important role in the heating of the solar chromosphere and corona (Biermann, 1948; Schwarzschild, 1948). Recent studies by Carlsson \textit{et al.} (2007) suggest that high frequency waves are not sufficient to heat the solar chromosphere.
Some other researchers (e.g. Cuntz et al., 2007; Wedemeyer-Bohm et al., 2007; Kalkofen, 2007) put in question these results and argue for high frequency waves to play an important role. There are assertions that stably stratified atmospheres can support and propagate not only acoustic waves, but also internal gravity waves. The significance of acoustic-gravity waves for the energy balance of the solar chromosphere is still being reconsidered. As a result, propagation of acoustic-gravity disturbances to the upper chromosphere, where these disturbances can be transformed into plasma disturbances and take part in the heating of the solar corona, is very important.

In this paper some new features of the acoustic-gravity wave propagation from the sources at the photospheric heights through a nonisothermal solar chromosphere are examined. For simplicity, we calculate the properties of the wave perturbation within the framework of a plane-layered model of the atmosphere. Taking into account the nonisothermicity of the solar atmosphere, we assume that the magnetic field in these regions of the solar atmosphere is relatively low on the corresponding spatial and temporal scales.

2. Basic Equations for Acoustic Gravity Waves in a Nonisothermal Atmosphere

In this section, we will shortly examine the equations for acoustic-gravity waves taking into account the height dependence of the temperature profile of the solar atmosphere. The linearized system of equations of gas dynamics for the pressure disturbance \( p \), the horizontal velocity \( u \), and the vertical velocity \( w \) is well known (Gossard and Hooke, 1975). Let \( z \) be the vertical coordinate and \( x \), the horizontal one:

\[
\begin{align*}
\frac{\rho_0}{\partial t} \frac{\partial u}{\partial t} &= - \frac{\partial p}{\partial x}, \\
\frac{\rho_0}{\partial t} \frac{\partial w}{\partial t} &= - \frac{\partial p}{\partial z} - \rho g, \\
\frac{\partial p}{\partial t} + w \frac{dp}{dz} &= c^2 \left( \frac{\partial \rho}{\partial t} + w \frac{d\rho}{dz} \right).
\end{align*}
\]

Here \( \vec{g} \) is the gravity acceleration, \( \rho_0 \) is the density of the equilibrium atmosphere, \( c \) is the adiabatic sound speed. In these equations, the regular
density \( \rho_0(z) = \frac{p_0(z)}{gH(z)} \). The regular pressure \( p_0 \) depends on height according to the thermodynamic equilibrium condition

\[
p_0(z) = p_{00} \exp \left[ - \int_0^z \frac{dz'}{H(z')} \right].
\] (2)

The sound speed \( c \) depends on height in a nonisothermal atmosphere. Indeed, the background temperature \( T \) depends on the vertical coordinate.

The following equation for the vertical velocity component can be obtained from system (1):

\[
c^2 \hat{Q} \frac{\partial^4 w}{\partial t^2 \partial z^2} + \gamma g \left[ c^2 \left( \frac{dH}{dz} + 1 \right) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{\partial^2}{\partial t^2} \right] \frac{\partial^3 w}{\partial t^2 \partial z} +
\]
\[
+ c^2 \gamma g^2 h \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w -
\]
\[
- \left[ \frac{\partial^2}{\partial t^2} \left( \hat{Q} \right)^2 + \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) g^2 (1 - \gamma) \hat{Q} \right] w = 0.
\] (3)

This equation has an exact solution for some model approximations of the atmospheric temperature profile (see, e.g., Savina, 1996). Below we examine the peculiarities of acoustic-gravity waves under conditions where the parentheses \( \hat{Q} = \left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} - c^2 \frac{\partial^2}{\partial y^2} \right) \) becomes equal to zero. We call the altitude at which this condition is fulfilled the resonance level.

The preliminary study of the peculiarities of acoustic-gravity wave propagation near this resonance level can be performed within the framework of the geometrical-optics approximation. Figure 1 shows a transformation of the dispersion dependence between a wave frequency and a horizontal wave number when the atmospheric temperature increases with altitude. In Figure 1 the solid curves correspond to the temperature \( T_1 \) and the broken curves correspond to the temperature \( T_2 \) \((T_2 > T_1)\). On the right panel in Figure 1 the regions of \( \omega \) and \( k_\perp \) for three regimes of the wave propagation are shown on a larger scale. The solution, obtained in this approximation contains two regimes (I or III, see Fig. 1), depending on the values of \( \omega \) and \( k_\perp \). In regime I, the waves partially reflect from the jump and partially pass into the upper medium. In regime III, the waves have internal reflectance with exponential damping in the upper medium. It is well known
that at an altitude level where the vertical wave number $k_z$ is equal to zero, the up propagating infrasonic wave reflects from the higher-temperature domain. Some energy leaks at a higher level which is close to the resonance level $z = z_*$, where $\omega - c(z_*)k_\perp = 0$. Regime II cannot be obtained in this approximation, because near the resonance level the vertical wave number is so small that the geometrical-optics approximation is not valid. This regime is analytically determined in the case of a smooth altitude temperature variation, and it is confirmed by the numerical calculation. We will show in the next section, that at the higher level than $z_*$, the wave field is absent. Thus, the wave propagation problem should be examined as a separated task.

3. Wave Perturbations near the Resonance Level

We analyze a more complicated model of acoustic-gravity waves in the chromosphere with a realistic altitude temperature profile. The linearized system of equations (1) for the wave perturbation can be reduced to the following form:

$$\begin{align*}
\left[-\omega^2 + \omega_g^2(z)\right] W - i\frac{\omega}{\rho_0} \left[\frac{\partial}{\partial z} + \Gamma(z)\right] P &= 0, \\
V &= \left(k_\perp/\omega\right) P, \\
\left[c^2(z)k_\perp^2 - \omega^2\right] P - i\omega\rho_0 c^2(z) \left[\frac{\partial}{\partial z} - \Gamma(z)\right] W &= 0.
\end{align*}$$

Figure 1: Dispersion relation of the acoustic-gravity wave for two different temperatures ($k_z = 0, T_2/T_1 = 2.5, K_\perp = k_\perp c_1/\omega_{g1}, \Omega = \omega_{g1}$).
Here, $V = (\rho/\rho_0)^{1/2}v$, $W = (\rho/\rho_{00})^{1/2}w$, and $P = (\rho_{00}/\rho_0)^{1/2}p$ are new variables, where $\rho_0$ and $\rho_{00}$ are the basic state densities in the current layer and at the bottom, respectively, $\omega_g$ is the Brunt-Väisälä frequency, and $\Gamma(z)$ is the Ekkard parameter. Field variables are proportional to $\exp(-i\omega t + ik_\perp x)$ for a monochromatic signal with frequency $\omega$ in plane atmospheric layers. In this model, the horizontal wave number $k_\perp$ is altitude independent and its main value is determined by the source scale. Let us consider in more detail the processes near the resonance level $z = z_*$, which are described by the system of equations

$$
\begin{align*}
\left[-\omega^2 + \omega_g^2(z_*)\right]W - i\frac{\omega}{\rho_{00}} \left[\frac{\partial}{\partial z} + \Gamma(z_*)\right]P &= 0, \\
\left[-\omega^2 + c^2(z)k_\perp^2\right]P - i\omega\rho_{00}c^2(z_*) \left[\frac{\partial}{\partial z} - \Gamma(z_*)\right]W &= 0.
\end{align*}
$$

(5)

Analytical study of this equations shows that for upward propagation at the level $z = z_*$ the conditions $W = 0$ and $\frac{dW}{dz} = 0$ are fulfilled (Savina and Bespalov, 2015; Bespalov and Savina, 2015). The absence of disturbances of both the vertical velocity and its derivative leads to the conclusion that above the level $z = z_*$ the solutions both for $W$ and for $P$ are identically equal to zero, as well as the averaged vertical energy flux. In order to counterbalance the pressure jump at the level in question, the finite mass should be concentrated at the resonance level, which is taken into account in the general solution by means of the delta function. Hence, if for the wave perturbation in the nonisothermal atmosphere at some level $z = z_*$ a condition $\omega = c(z_*)k_\perp$ is satisfied, then the averaged vertical energy flux is equal to zero, and above the first of such levels, wave perturbations are absent along the vertical propagation path. Under the real conditions, the resonance in the form of the delta function in the pressure disturbance, and in the horizontal velocity ($V = P/c(z = z_*)\rho_0$), is limited by dissipation and nonlinearity.

4. Results of Calculations for Resonance Perturbation in the Solar Chromosphere

In this Section, we determine perturbations of the pressure and vertical velocity by means of numerical calculations. We assume that at the photosphere level there is a monochromatic source of vertical velocity and that at...
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Figure 2: Temperature height profile of the solar atmosphere.

Figure 3: Results of numerical calculation.

The altitudes higher than the transition region the atmosphere is isothermal (upper boundary condition). The temperature height profile conforms to the model (Vernazza et al., 1981) and is marked by asterisks in Figure 2. The temperature spline profiles are approximated by a tenth-order polynomial (a solid curve).

The wave fields are conveniently calculated in dimensionless variables, as which we chose \( \tilde{\omega} = \omega/\omega_0 \), \( \tilde{k}_\perp = k_\perp c/\omega_0 \), \( \tilde{c} = c(z)/c_0 \), \( \tilde{W} = W/W_0 \), and \( \tilde{P} = P/\rho_0 c_0^2 \). Here, the subscript "0" indicates the value of the variable on the bottom chromosphere.

The results of numerical calculation using wave equation in Riccaty form (Savina et al., 2006) are given in Figure 3 for \( \tilde{\omega} = 1.7 \) and \( \tilde{k}_\perp = 1.7 \). The upper panel shows the height dependence \( \tilde{c}(z)\tilde{k}_\perp \) and the straight line is
the frequency $\tilde{\omega}$. On the bottom panel, the solid curves are the altitude dependence of the pressure amplitude $|\tilde{P}(z)|$ and the dashed curves are the altitude dependence of the vertical velocity amplitude $|\tilde{W}(z)|$.

Thus, we have shown that if the resonance condition $\omega = c(z_*) k_{\perp}$ is satisfied at the level $z = z_*$, then above this level the wave fields and average energy flow are absent. As the analysis shows, this regime is absent for the two-layer model with different temperature.

5. Conclusions

The wave component of pressure has singularities near the altitude $z_*$, where the horizontal phase velocity is equal to the local sound speed. The vertical velocity and its derivative in perturbation turn to zero at this altitude. The wave perturbations are absent above this altitude. The dissipation and nonlinearity limit the pressure wave singularities.

Three regimes of acoustic-gravity waves propagation in the solar atmosphere are possible. Partial passage through the temperature profile is realized with a fixed value of the horizontal wave number for sufficiently high frequencies (for horizontal scales of about 1000 km, the wave periods can be smaller than 50 s). Internal reflectance is realized at the lower frequencies. In this case, the amplitude of wave field falls exponentially, but it is comparatively slow because of the high value of the pressure scale height, $H$. Then the wave perturbations can transform into a plasma perturbation, which can heat the corona. The third new regime is realized if resonance condition is satisfied inside the chromosphere. Then the temperature discontinuity plays the role of a solid wall, while at a higher level the wave field is absent.

The effect examined can be realized for the fast branch of acoustic-gravity waves. For the actual temperature profile of the solar atmosphere a filtration of waves is possible in the transition region of solar atmosphere for horizontal scales of the order of 4000 km or less. Thus, the energy of a relatively high frequency acoustic gravity waves with period less then two minutes cannot leak to the upper chromosphere, and these waves do not contribute to the vertical energy flux to the corona. A special feature of the pressure field and horizontal speed, which appears in the resonance region, can cause the generation of plasma waves, penetrating into the corona.

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