Coronal and Prominence Diagnostics based on Transverse Oscillations: Analytic Approach

Bojan Vršnak

Hvar Observatory, Croatia

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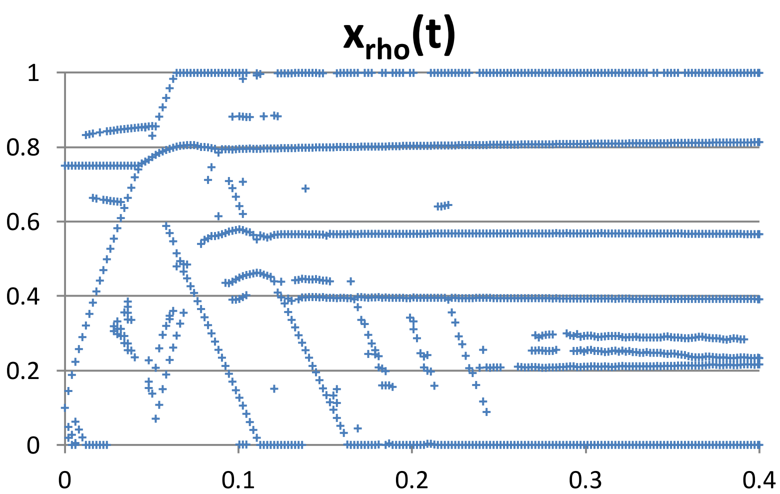
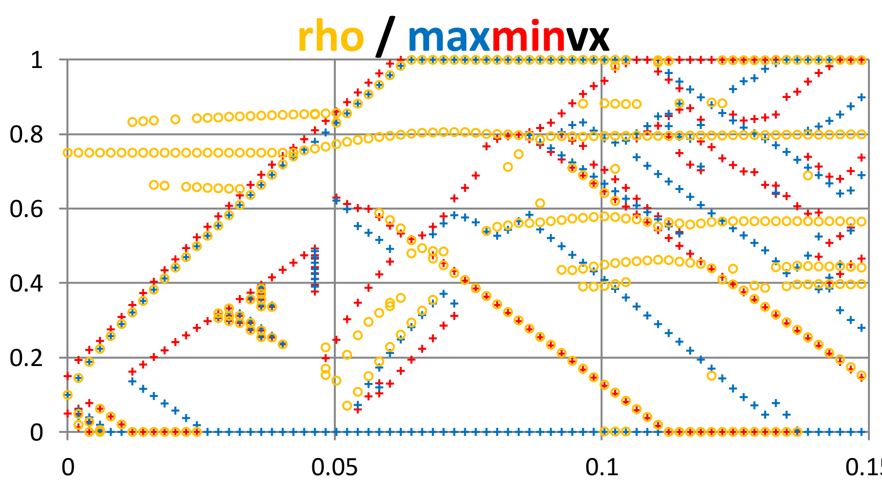
Simple Wave –> Dense Obstacle

(prominence, streamer, pseudo-streamer)

free

fixed





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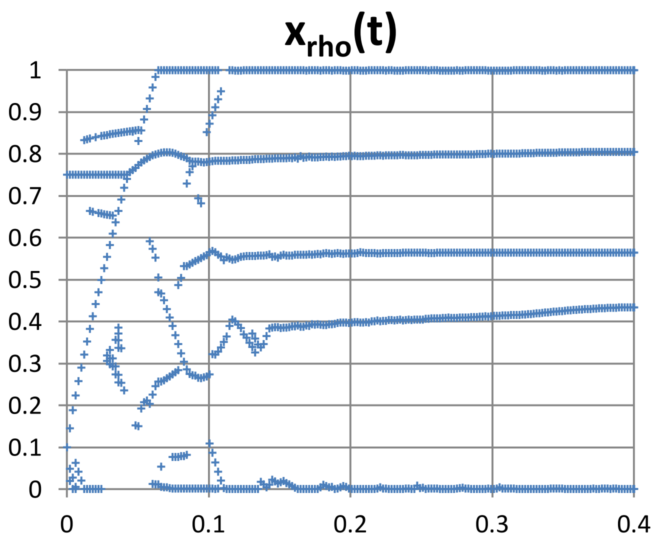
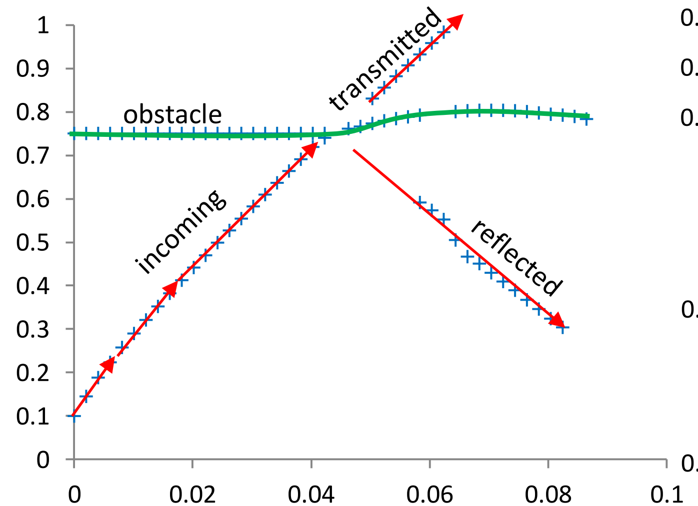
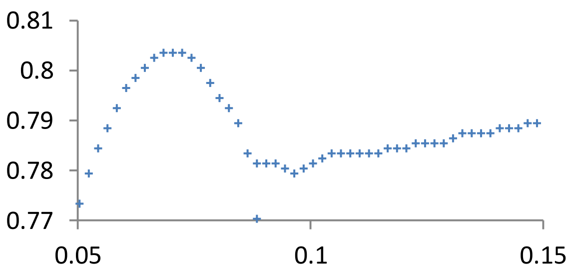
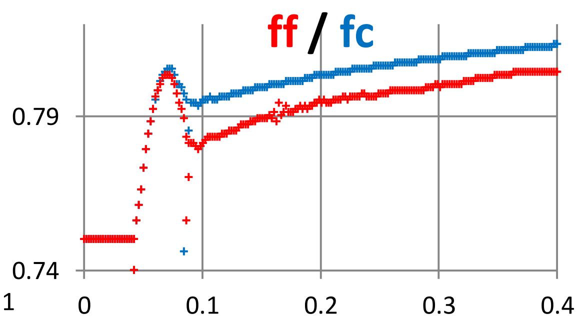
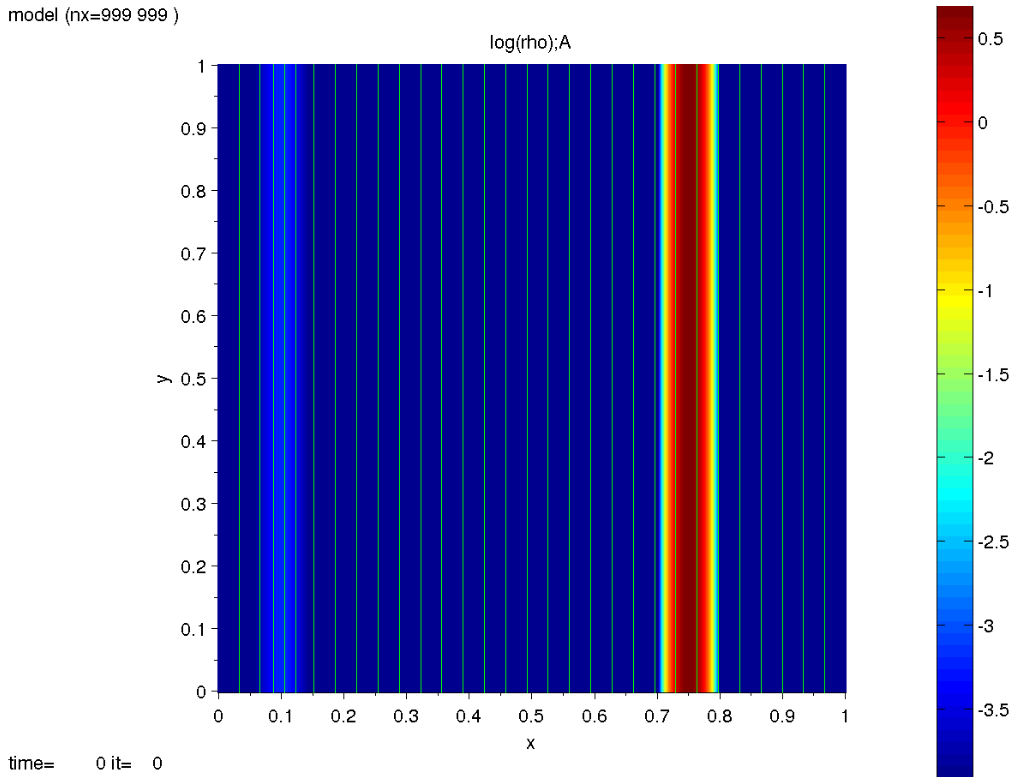


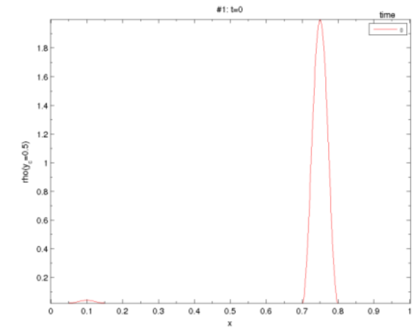


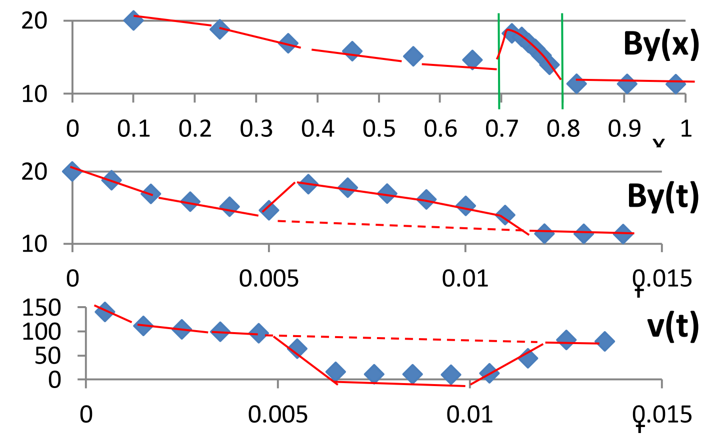


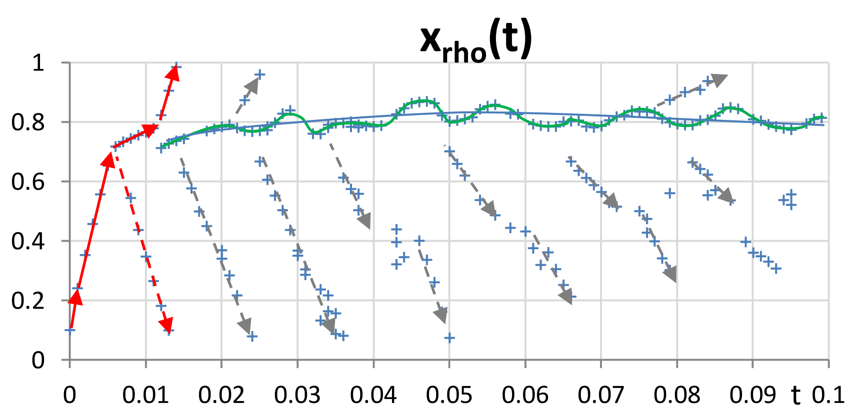


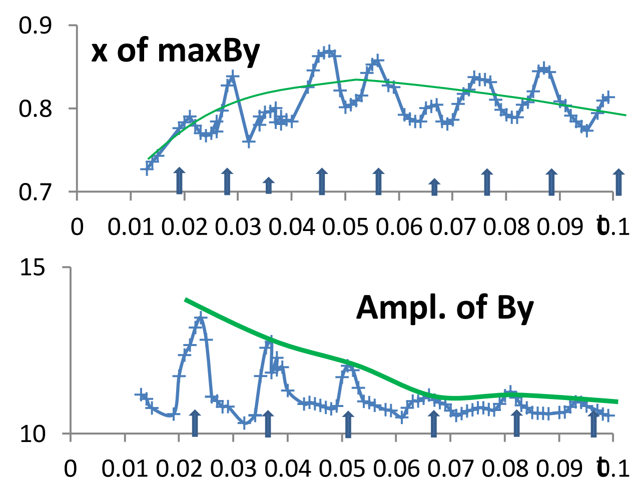
Simple Wave –> Dense Obstacle









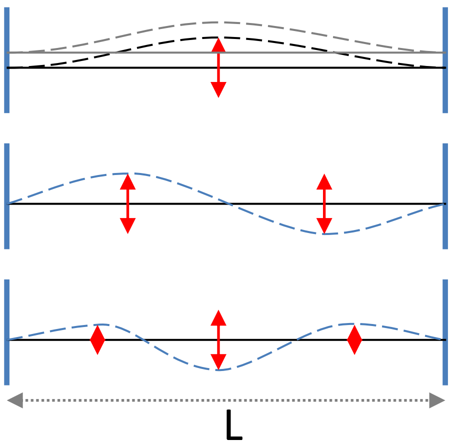


Analytical Model: Basic Modes

$$a(x,t) = -c_1 x + c_2 f(t) - c_3 v$$

$[\ c_1 = \omega^2\ ;\ \ c_3 = 2\gamma\ ]$

"kink" mode



$\omega_0 = \omega_k$

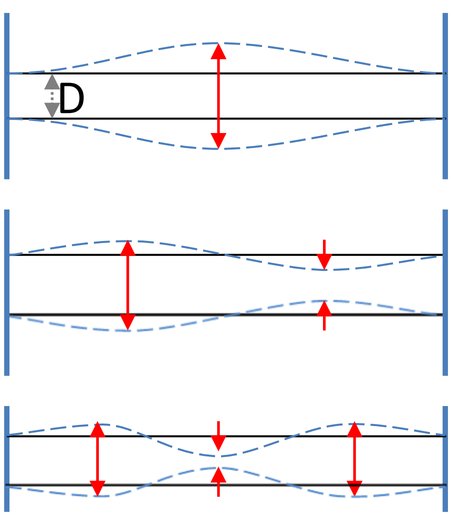
$\omega_{k2} = \omega_k/2$

$\omega_{k3} = \omega_k/3$

$c_1 \sim B^2/\mu_0 \rho R \sim 2xB^2/\mu_0 \rho L^2$

$\omega_k \sim v_{A0}/L$ -> $T_k \sim 2\pi L/v_{A0}$

"sausage" mode



$\omega_0 = \omega_s$

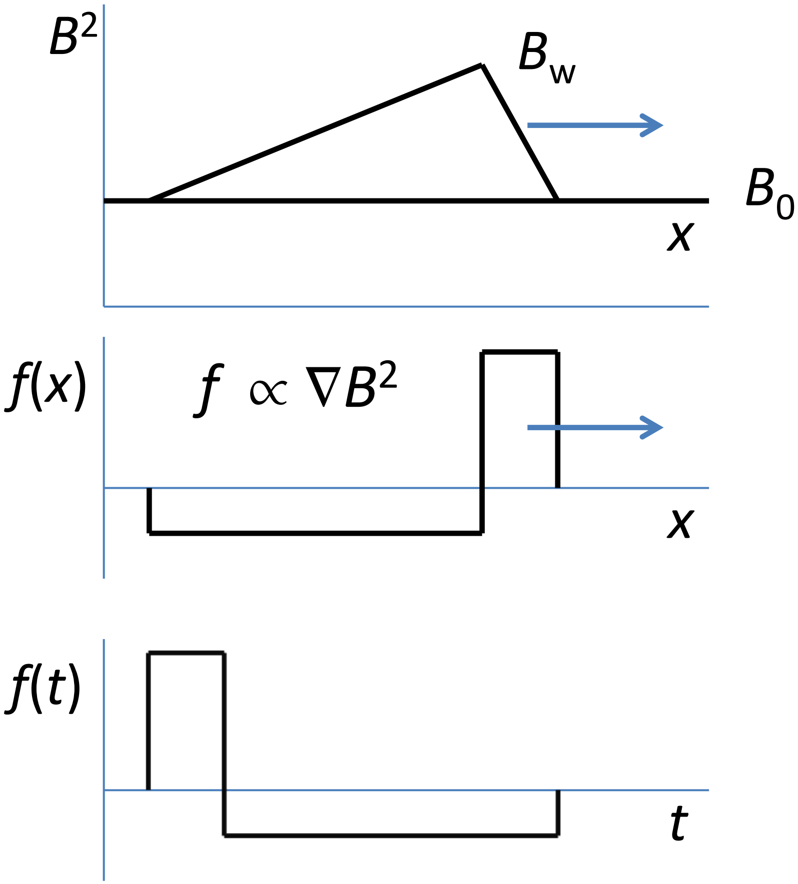
$\omega_{s2} = \omega_s/2$

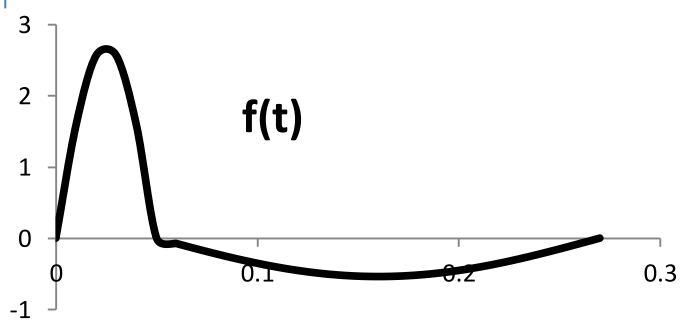
$\omega_{s3} = \omega_s/3$

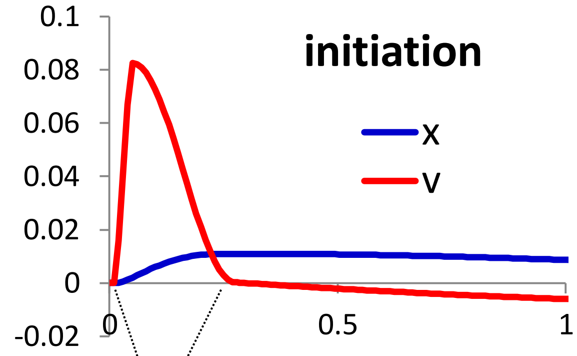
Similarly: $T_s \sim 2\pi D/v_{A0}$

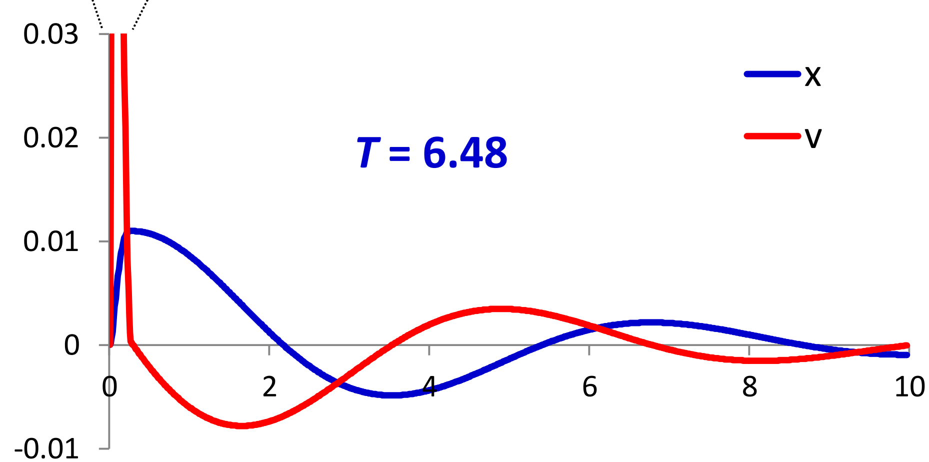
$T_k/T_s \sim L/D$

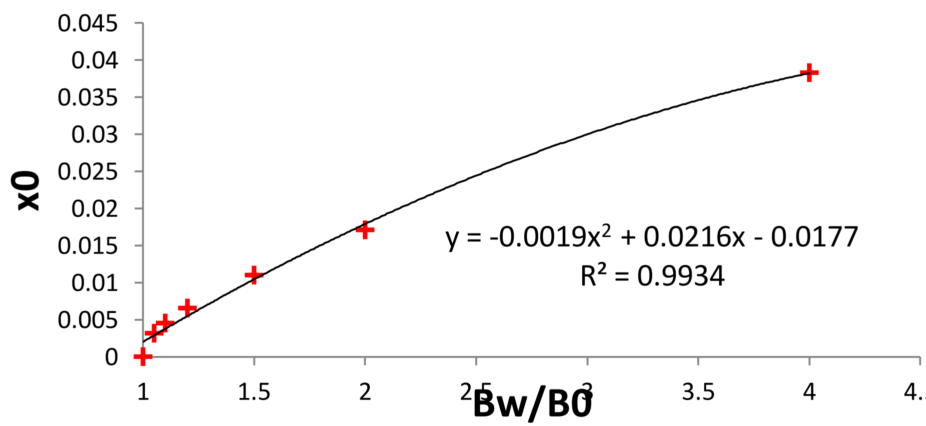
Analytical model: Oscillation Triggering











Wave -> Obstacle

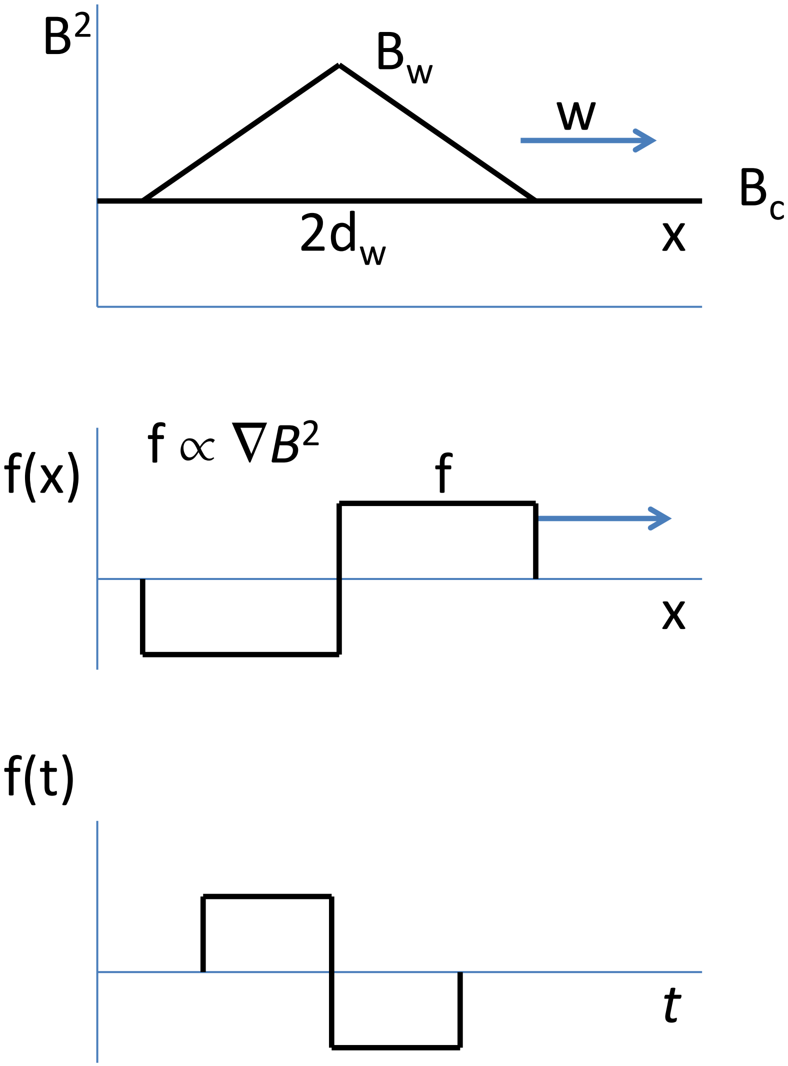
Damped Oscillator:

$$\ddot{x} + \omega^2 x + 2m\delta\dot{x}(t) + f(t) = 0$$

Key parameters:

- maximum speed v_m
- acceleration length: $x(v_m) = x_1$
- acceleration time t_1
- initial acceleration a_0
- period: P
- amplitude x: $x_m = x_2$
- [damping: δ]

Analytical model: Oscillation Triggering



(neglecting damping)

Eq 1a: $\rho_p v_m^2/2 = \int_0^{x1} (f - \omega^2 x)\,dx$

[Eq 1b: $\rho_p v_m^2/2 = \int_{x1}^{x2} (f + \omega^2 x)\,dx$]

Eq 2: $w_m = V_{Ap} + 3v_m/2$

$\omega = 2\pi/P = 2\sqrt{2}\, V_{Ap} / \lambda$

$f = \rho_p a_0 = (B_{wp}^2 - B_p^2) / 2\mu d_{wp}$

Results: Prominence

$$\omega^2 = (2\pi/P)^2 = 8V_{Ap}^2 / \lambda^2, \quad \text{i.e.,} \quad \boxed{V_{Ap} = \pi^2\lambda/2P}$$

Eq 1: $\boxed{\rho_p = \omega^2 x_1^2 / (2a_0x_1 - v_m^2)}$ & $\boxed{w_{pm} = V_{Ap} + 3v_m/2}$

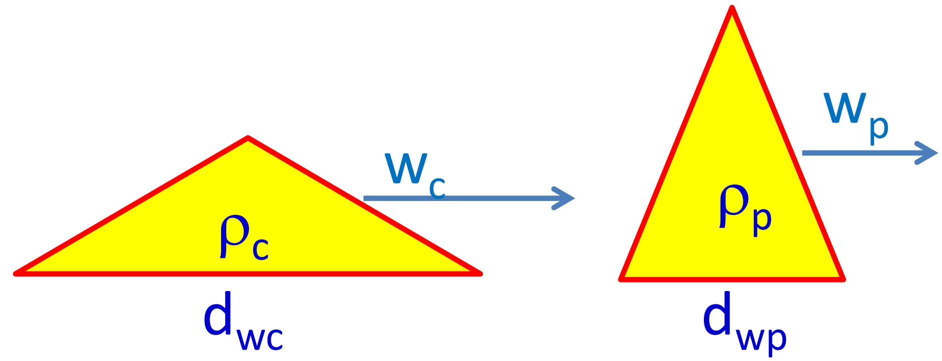
i.e., $\boxed{B_p = V_{Ap}(\mu\rho_p)^{1/2}}$ & $\boxed{f = \rho_p a_0}$

Eq 2 (for $d_w >> d_p$) : $\boxed{d_{wp} = (V_{Ap} + 3v_m/2)\, t_1}$

$$f = (B_{wp}^2 - B_p^2) / 2\mu d_{wp} = (X_p^2 - 1)\, B_p^2 / 2\mu d_{wp}$$

$\boxed{B_{wp}^2 = B_p^2 + 2\mu f d_{wp}}$, i.e., $\boxed{X_p^2 = 1 + 2\mu f d_{wp}/B_p^2}$

Results: Corona



(for $d_w >> d_p$): $d_{wp}/w_p = d_{wc}/w_c$, i.e., $d_{wc} = d_{wp} w_c / w_p$

$\Delta\rho_p \, d_{wp} = \Delta\rho_c \, d_{wc}$, i.e., $(\rho_c/\rho_p)(X_c - 1)/(X_p - 1) = w_p/w_c$

$(w_c^2 - V_{Ac}^2) = (w_p^2 - V_{Ap}^2) \, w_p/w_c$, i.e.,

$$(V_{Ac}/V_{Ap})^2 = (w_c/V_{Ap})^2 - [(w_p/V_{Ap})^2 - 1] \, (w_p/w_c)$$

&

$$\rho_c/\rho_p = (V_{Ac}/V_{Ap})^2$$

&

$$B_c = V_{ac} \, (\mu\rho_c)^{1/2}$$