Reflection of Cosmic Rays at Oblique MHD Shocks

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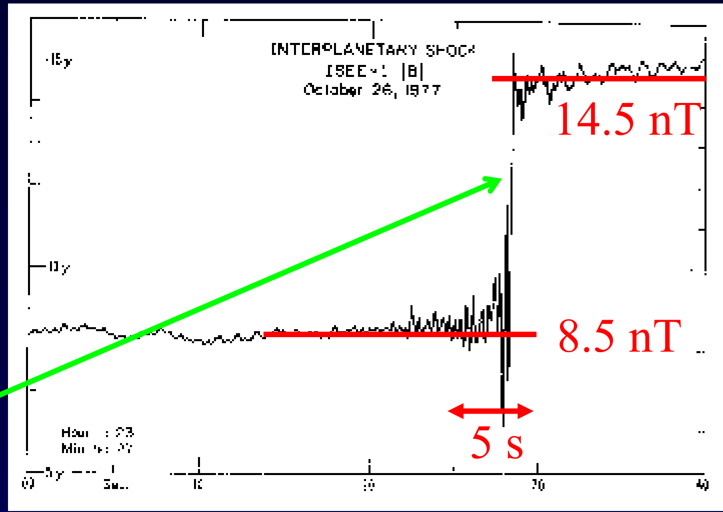
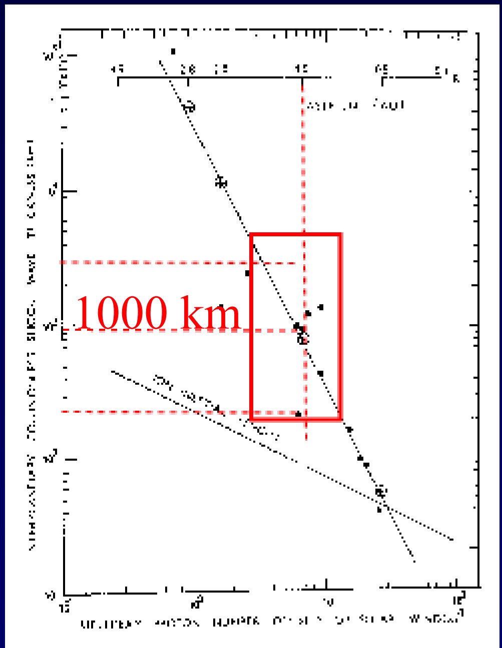
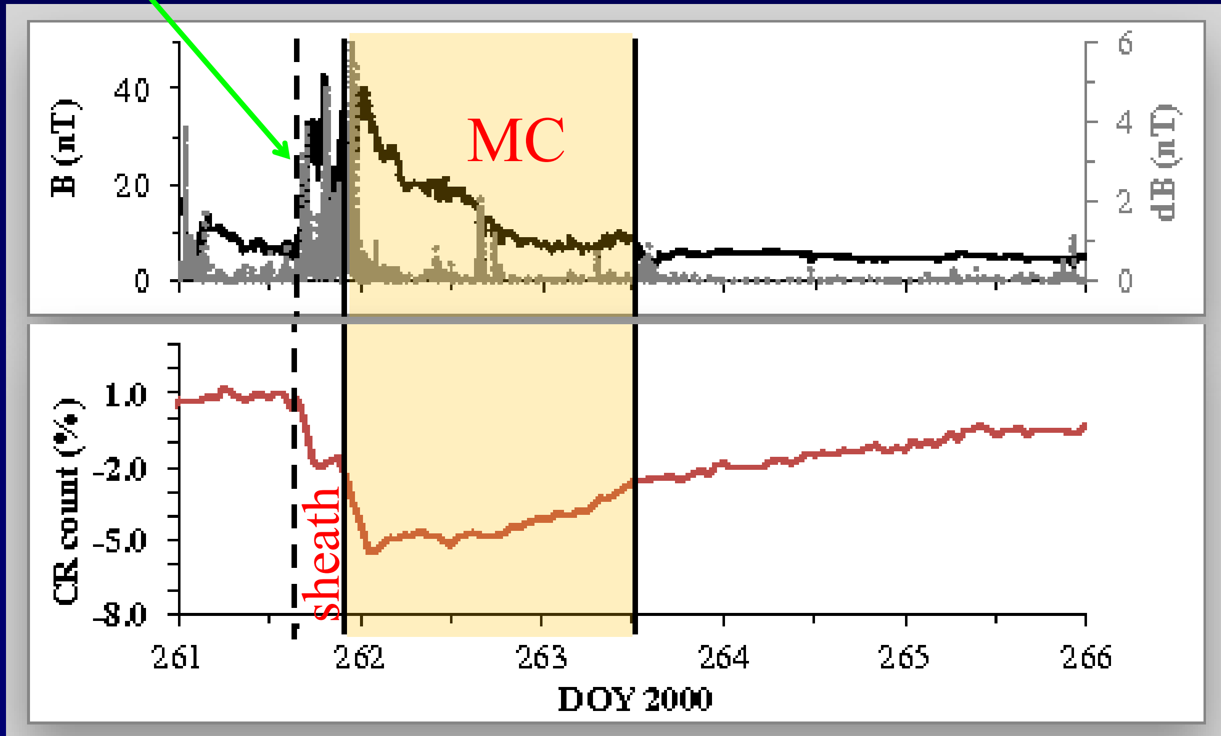
Introduction



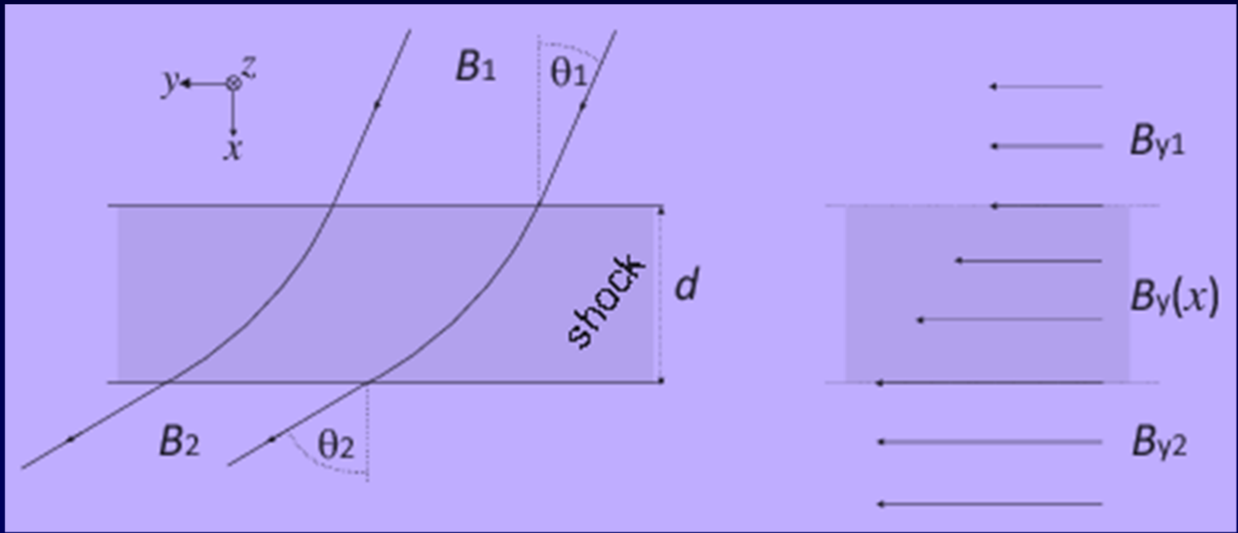


shock



The Model

oblique 2D MHD shock
(z-invariant, B_z=0)



Pinter 1980:

d ~ 1000 km

i.e.

$d \ll r_L$

$\Delta t(d) \ll t_{cycl}$

flux conservation: $B_{x1} = B_{x2}$ => $B_x(x)$ = const. , i.e., $\partial B_x/\partial x = 0$;

divB=0 => $\partial B_y/\partial y = 0$, i.e., $B_y(y)$ = const.

<u>approximation:</u>

$B_y(x) = B_{y1} + kx$ ($k = \Delta B_y/d$, $\Delta B_y = B_{y2} - B_{y1}$)

Particle motion

$d \ll r_L$, $\Delta t(d) \ll t_{cycl}$

=> ~~guiding centre approx.,~~
~~magnetic moment conservation~~

incoming particle: v_x , v_y , v_z

Lorentz force: $m\boldsymbol{a} = q\,\boldsymbol{v} \times \boldsymbol{B}$; $m = \gamma m_0$

$a_x = -\,q/m\; v_z B_y$

$a_y = q/m\; v_z B_x$

$a_z = q/m\; (v_x B_y - v_y B_x)$

$B_x = \text{const.}$

$B_y = B_{y1} + kx$

“Components” of Particle Motion

$a_x = -q/m\,[\,v_z B_{y1} + v_z \Delta B_y(x)\,]$

$a_y = q/m\, v_z B_x$

$a_z = q/m\,[\,v_x B_{y1} + v_x \Delta B_y(x) - v_y B_x\,]$

$\Delta B_y(x) = \Delta B_y\, x/d$

$\Delta B_y = B_{y2} - B_{y1}$

“components”:

- cyclotron rotation related to B_1 (written in a coordinate system inclined by θ_1 from the B_1 direction)
- ΔB_y effect: decelerates/accelerates particle in x-direction, accelerates/decelerates particle in z-direction ($v_x^2 + v_z^2$ = const.)

ΔB_y effect = shock effect

$$a_x \equiv \dot{v}_x = -q/m \; v_z \, \Delta B_y(x)$$

$$a_z \equiv \dot{v}_z = q/m \; v_x \, \Delta B_y(x)$$

(magnetic-mirror “equivalent”)

the system can be solved numerically for a given function $\Delta B_y(x)$

<u>or analytically in an approximative form:</u>

a=const. approximation:

1. $a_x(x) \rightarrow \{a_x\} = -q/m \, \{v_z\}\{\Delta B_y\}$
2. $a_z(x) \rightarrow \{a_z\} = q/m \, \{v_x\}\{\Delta B_y\}$

$$\{\Delta B_y\} = (B_{y2} - B_{y1})/2 =$$

$$= (B_{y2}/B_{y1} - 1) \, B_{y1}/2 =$$

$$= b \; B_{y1}/2$$

Reflection ($v_{x2} = 0$)

$x(t) = v_{x1}\, t + \{a_x\}\, t^2/2$, $v_x(t) = v_{x1} - \{a_x\}\, t$

reflection if: $v_x = 0$ at $x = x_0 < d$

i.e., if the stopping distance $x_0 = v_{x1}^2 / 2\{a_x\}$ is $x_0 < d$

all particles with $v_{x1} < (2\{a_x\}d)^{1/2}$ are reflected

=> for a given $\{a_x\} = f(v_{z1})$,

find critical v_{x1} for which $x_0 = d$

$\mathcal{B} = B_{y2}/B_{y1} - 1$

$\{v_x\} = v_{x1}/2$

$\Rightarrow \{a_z\} = q/m\,\{v_x\}\{\Delta B_y\} = q/m\, v_{x1} \mathcal{B} B_{y1} d/4$

$v_{z2} = v_{z1} + \{a_z\}t_d = v_{z1} + \{a_z\}d/\{v_x\} = v_{z1} + 2\{a_z\}d/v_{x1}$

$\{v_z\} = (v_{z1} + v_{z2})/2 = \ldots = v_{z1} + q/m\, \mathcal{B} B_{y1} d/4$

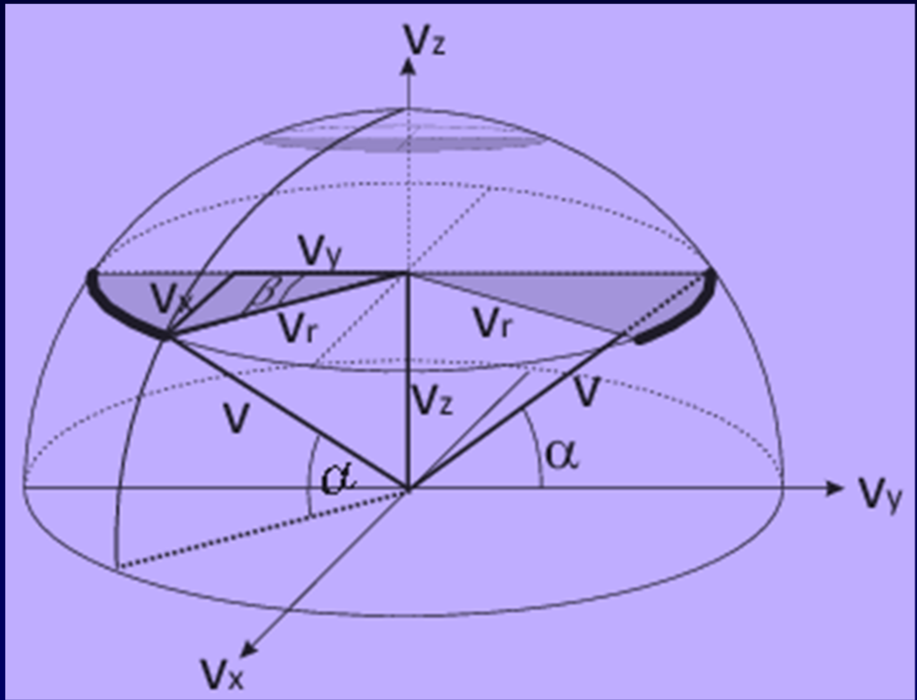
$\{a_x\} = q/m\, \{v_z\}\, \{\Delta B_y\} = q/m\, (v_{z1} + q/m\, \mathcal{B} B_{y1} d/4)\, \mathcal{B} B_{y1}/2$

finally, we get the equation determining critical v_{x1}:

$$v_{x1}^2 = q/m\, (v_{z1} + \boxed{q/m\, \mathcal{B} B_{y1} d/4})\, \mathcal{B} B_{y1}\, d$$

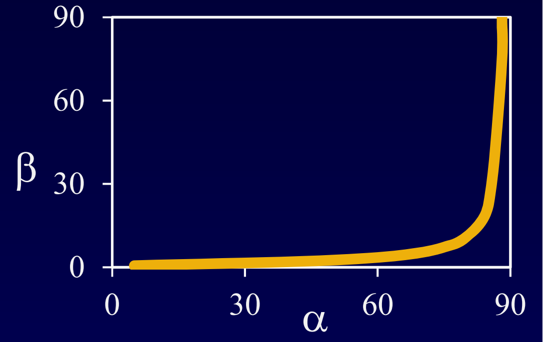
$$= 4A\,(v_{z1} + A)$$

Integral over the Velocity Space



$$\sin\alpha = v_z/v$$

$$\cos\alpha = v_r/v$$



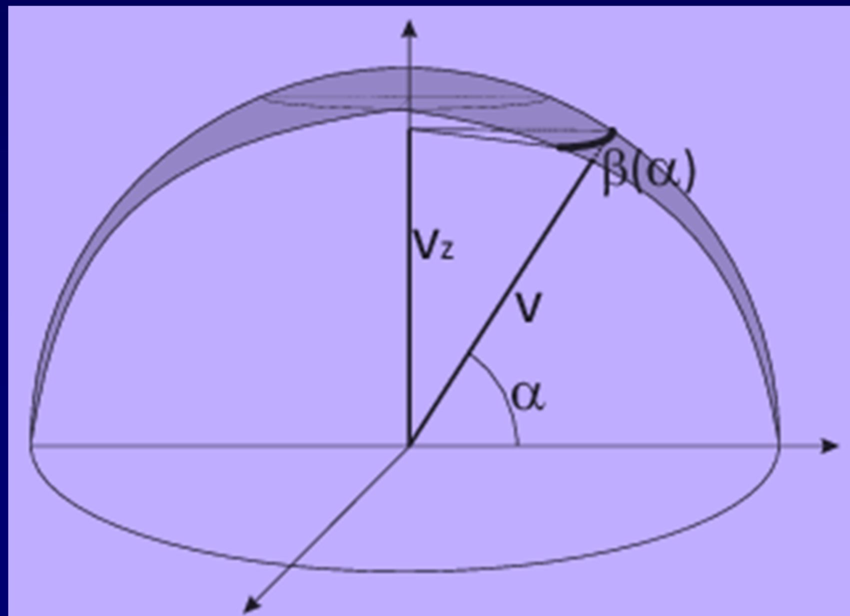
$$\sin\beta = v_x/v_r$$

$$= v_x/v\cos\alpha$$

$$= (4A(\sin\alpha + A/v)/v)^{1/2} / \cos\alpha$$

$$\mathcal{A}_1 = \int_0^{\alpha_0}\int_0^{\beta(\alpha)} \cos\alpha \, d\alpha \, d\beta \;/\pi$$

$$\mathcal{A}_2 = \int_0^{2\pi}\int_{\alpha_0}^{\pi/2} \cos\alpha \, d\alpha \, d\beta \;/2\pi$$



$$\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2$$

Integral over the Velocity Space

for a given v_{z1} , particles with
$v_{x1}^{2} < 4A\,(v_{z1} + A)$ are reflected

$[\ A = q/m\ \mathcal{B} B_{y1} d/4\ ]$

limits:

1. $v_{x1} > 0$
2. $v_{z1} > -A$
3. for a given v_{z1} , the largest v_{x1} (i.e., when v_{y1}=0) is defined by $v_{x1}^{2} + v_{z1}^{2} = v^2$

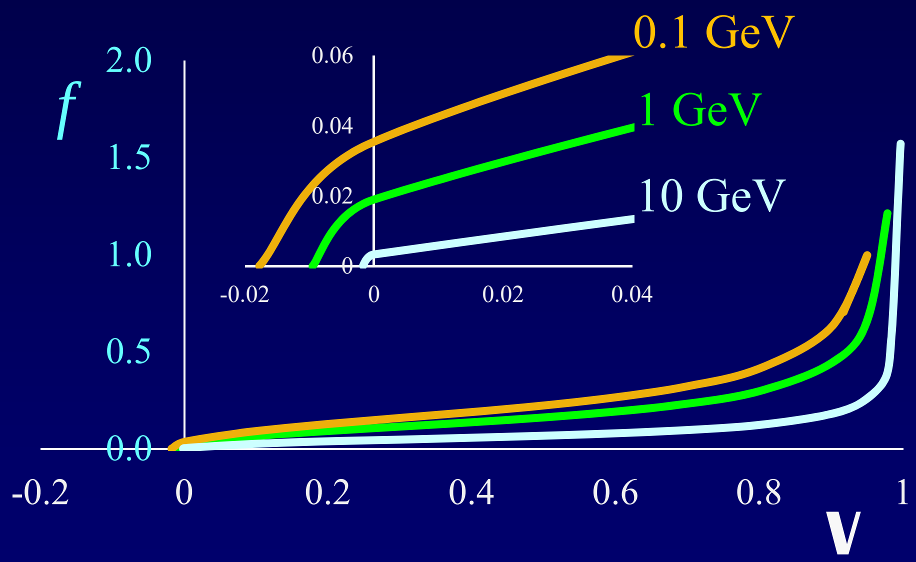
$$v^2 - v_{zc}^{2} = 4A\,(v_{zc} + A) \quad \text{(for } v_{z1} > v_{zc} \text{ all reflected)}$$

$$v_{zc} = -\,2Av_{zc} + [\ 4A^2 v_{zc}^{2} - (4A^2 - v^2)\ ]^{1/2}$$

Integral over the Velocity Space

$$\mathcal{A}_1 = 1/\pi \int_{\mathbf{V}_0}^{\mathbf{V}_c} \operatorname{asin}\left[\left((\mathbf{V}v/A + 1) / (1 - \mathbf{V}^2) \right)^{1/2} 2A/v \right] d\mathbf{V}$$

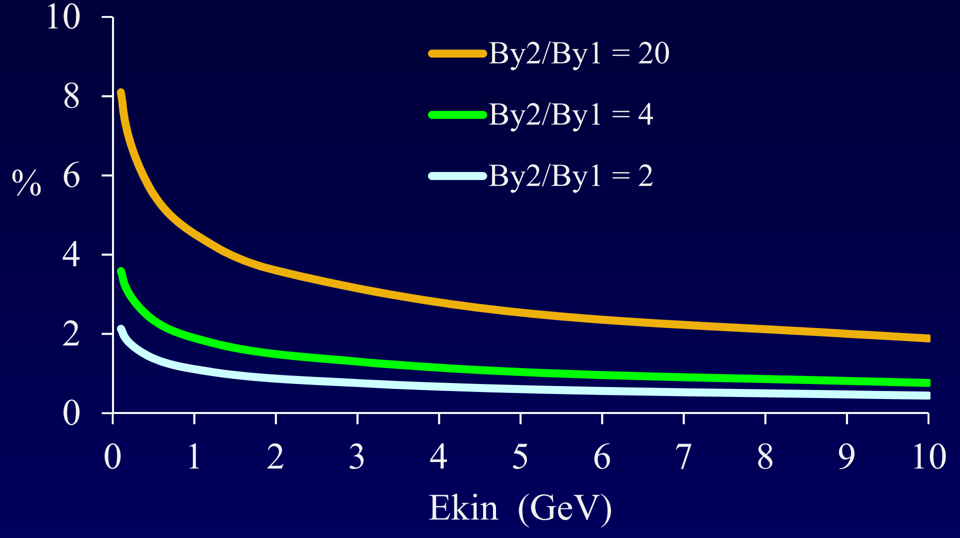
$$(\mathbf{V} = v_z/v \quad ; \quad \mathbf{V}_0 = -A/v \quad ; \quad \mathbf{V}_c = v_{zc}/v)$$

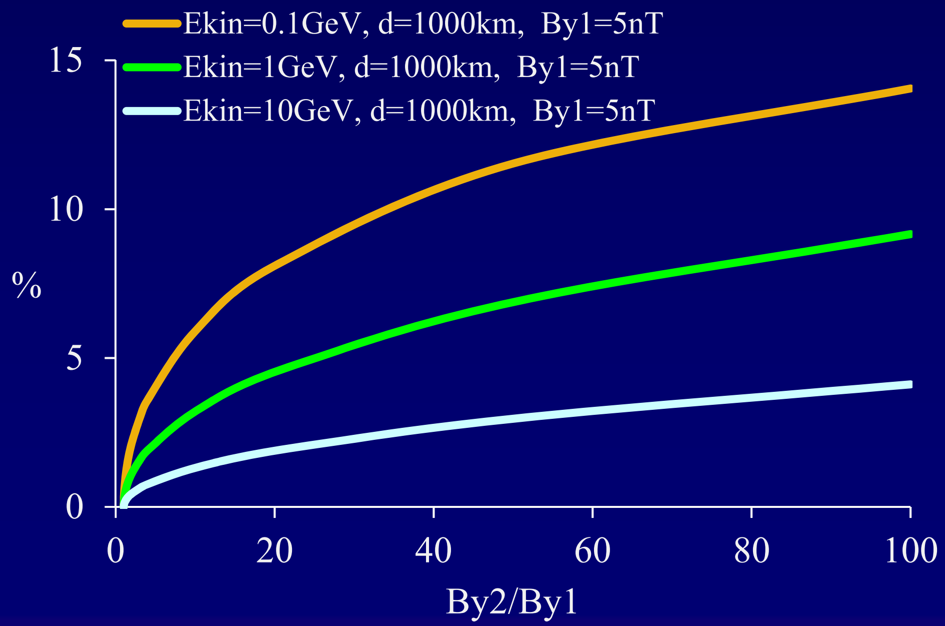


$$\mathcal{A}_2 = (1 - v_{zc}/v) /2$$

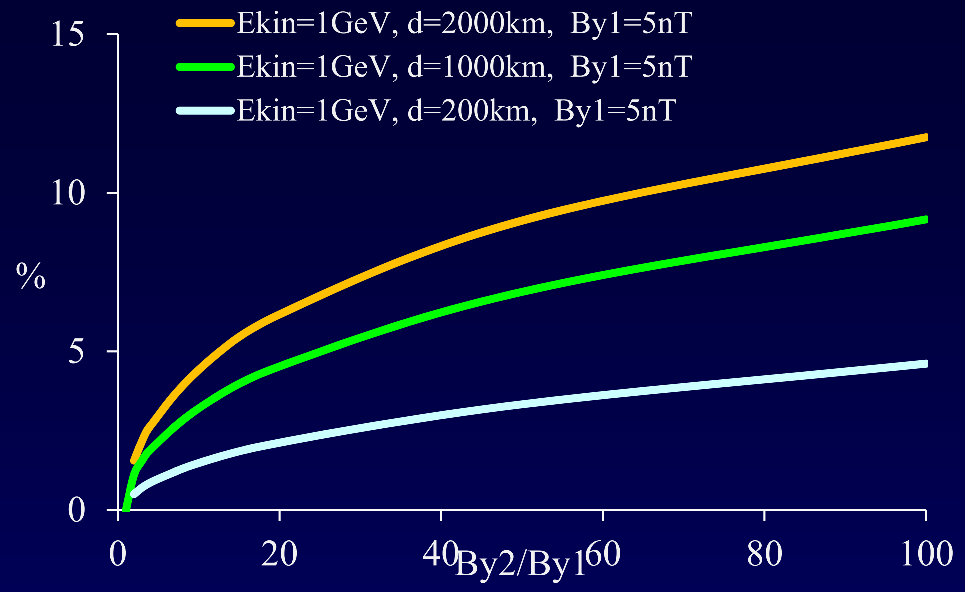
$$\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 \quad (\mathcal{A}_2 \ll \mathcal{A}_1)$$

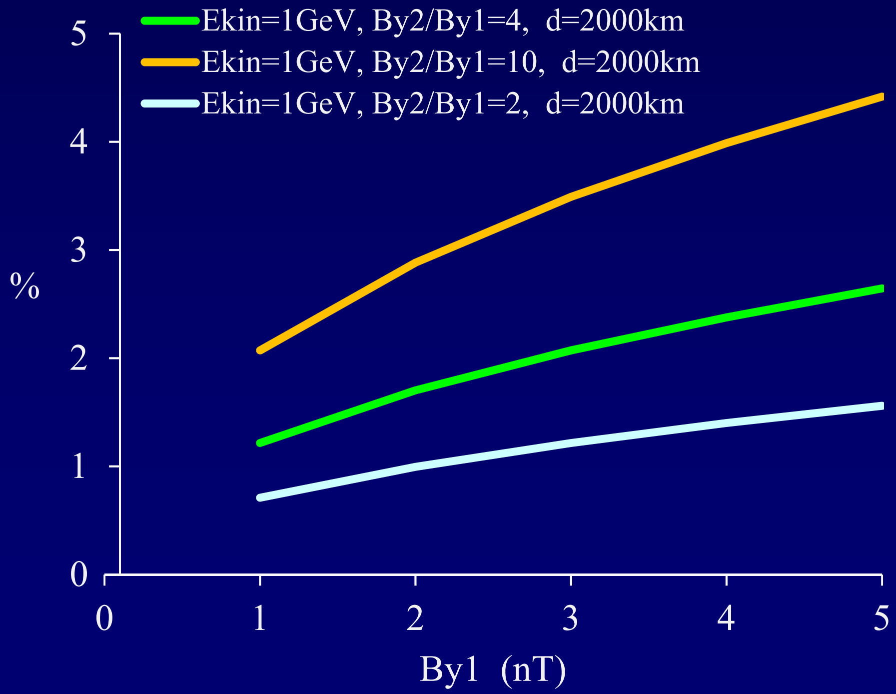
Results: Energy Dependence

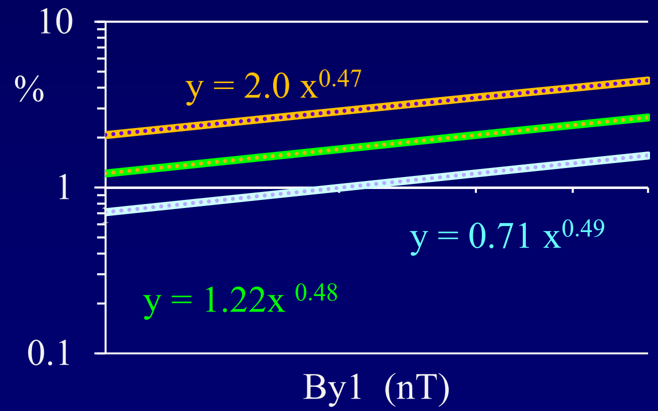




Results for E_{kin} = 1 Gev







$\mathcal{A} \sim B_{y1}^{1/2}$; $\mathcal{A} \sim d^{1/2}$

Thank you for your attention !