Coronal and Prominence Diagnostics based on Transverse Oscillations: Analytic Approach

Bojan Vršnak

Hvar Observatory, Croatia

This work has been fully supported by Croatian Scientific Fundation under the project 6212 „Solar and Stellar Variability“ (SOLSTEL).
Simple Wave $\rightarrow$ Dense Obstacle

(prominence, streamer, pseudo-streamer)
Simple Wave $\rightarrow$ Dense Obstacle

\[ x_{\rho}(t) \]

\[ \text{fixed} \]

\[ \text{fixed} \]

\[ \text{obstacle} \]
Simple Wave  \rightarrow\text{Dense Obstacle}
Analytical Model: Basic Modes

“kink” mode

\[ a(x,t) = -c_1 x + c_2 f(t) - c_3 v \]

\[ [c_1 = \omega^2; \ c_3 = 2\gamma] \]

\[ \omega_0 = \omega_k \]

\[ \omega_{k2} = \omega_k / 2 \]

\[ \omega_{k3} = \omega_k / 3 \]

“sausage” mode

\[ \omega_0 = \omega_s \]

\[ \omega_{s2} = \omega_s / 2 \]

\[ \omega_{s3} = \omega_s / 3 \]

\[ c_1 \sim B^2/\mu_0\rho R \sim 2xB^2/\mu_0\rho L^2 \]

\[ \omega_k \sim v_{A0}/L \rightarrow T_k \sim 2\pi L/v_{A0} \]

Similarly: \( T_s \sim 2\pi D/v_{A0} \)

\[ T_k/T_s \sim L/D \]
Analytical model: Oscillation Triggering

\[ f(x) \propto \nabla B^2 \]

\[ f(t) \]

\[ y = -0.0019x^2 + 0.0216x - 0.0177 \]

\[ R^2 = 0.9934 \]
Wave -> Obstacle

Damped Oscillator:

\[ \ddot{x} + \omega^2 x + 2m\delta \dot{x}(t) + f(t) = 0 \]

Key parameters:
- maximum speed \( v_m \)
- acceleration length: \( x(v_m) = x_1 \)
- acceleration time \( t_1 \)
- initial acceleration \( a_0 \)
- period: \( P \)
- amplitude \( x \): \( x_m = x_2 \)
- [damping: \( \delta \)]
Analytical model: Oscillation Triggering

(neglecting damping)

Eq 1a: \[ \frac{\rho_p v_m^2}{2} = \int_0^{x_1} (f-\omega^2 x)dx \]

[ Eq 1b: \[ \frac{\rho_p v_m^2}{2} = \int_{x_1}^{x_2} (f+\omega^2 x)dx \] ]

Eq 2: \[ w_m = V_{Ap} + 3v_m/2 \]

\[ \omega = \frac{2\pi}{P} = 2\sqrt{2} \frac{V_{Ap}}{\lambda} \]

\[ f = \rho_p a_0 = \left( B_{wp}^2 - B_p^2 \right) / 2\mu d_{wp} \]
Results: Prominence

\( \omega^2 = \left( \frac{2\pi}{P} \right)^2 = 8V_{Ap}^2 / \lambda^2 \), i.e., \( V_{Ap} = \frac{\pi^2 \lambda}{2P} \)

Eq 1: \( \rho_p = \frac{\omega^2 x_1^2}{(2a_0 x_1 - v_m^2)} \) & \( w_{pm} = V_{Ap} + 3v_m/2 \)

i.e., \( B_p = V_{Ap} \left( \mu \rho_p \right)^{1/2} \) & \( f = \rho_p a_0 \)

Eq 2 (for \( d_w >> d_p \)): \( d_{wp} = (V_{Ap} + 3v_m/2) t_1 \)

\( f = \frac{(B_{wp}^2 - B_p^2)}{2\mu d_{wp}} = \frac{(X_p^2 - 1) B_p^2}{2\mu d_{wp}} \)

\( B_{wp}^2 = B_p^2 + 2\mu fd_{wp} \), i.e., \( X_p^2 = 1 + 2\mu fd_{wp}/B_p^2 \)
Results: Corona

(for \(d_w >> d_p\)): \(d_{wp}/w_p = d_{wc}/w_c\), i.e., \(d_{wc} = d_{wp}w_c/w_p\)

\[\Delta \rho_p d_{wp} = \Delta \rho_c d_{wc}, \ i.e., \ \left(\frac{\rho_c}{\rho_p}\right)\left(X_c - 1\right)/\left(X_p - 1\right) = w_p/w_c\]

\[\left(w_c^2 - V_{Ac}^2\right) = \left(w_p^2 - V_{Ap}^2\right) w_p/w_c, \ i.e., \]

\[\left(\frac{V_{Ac}}{V_{Ap}}\right)^2 = \left(\frac{w_c}{V_{Ap}}\right)^2 - \left[\left(\frac{w_p}{V_{Ap}}\right)^2 - 1\right] \left(\frac{w_p}{w_c}\right)\]

\[\rho_c/\rho_p = \left(\frac{V_{Ac}}{V_{Ap}}\right)^2\]

\[B_c = V_{ac} (\mu \rho_c)^{1/2}\]